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Relaxing Axions

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Abstract

A mechanism for lifting the cosmological upper bound on the axion decay constant, f_a , is proposed. It entails the near masslessness of the radial mode whose vacuum expectation value is f_a . Energy in the coherent oscillations of the axion field in the early universe gets fed into the motion of the radial mode, from which it is then redshifted away. It is found that the initial value of f_a can be at scales between 2×10^{14} GeV and 10^{16} GeV. This evolves with time to values close to the Planck scale. It is suggested that the nearly massless radial mode might play the role of quintessence.

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1 Introduction

The axion¹ idea is an extremely elegant approach to solving the Strong CP Problem. However, it is not without difficulties. The main difficulty is that axions have neither been observed directly in the laboratory, nor indirectly through astrophysical effects. This implies that the axion decay constant, f_a , must be greater than about 10^{10} GeV. Moreover, there is the cosmological bound, based on the so-called “axion energy problem”,² that f_a is less than about 10^{12} GeV. This means that the axion decay constant (or, equivalently, the scale at which the Peccei-Quinn symmetry is broken) cannot be at any of the mass scales at which other physics is known or suspected to be based. In particular, it cannot be at the weak scale, the Planck scale, or the grand unification scale.

A number of attempts³ have been made to weaken or remove the cosmological bound on f_a . In this paper we suggest an idea for doing this. The idea is that if the radial mode associated with the axion (that is, the scalar field whose expectation value is f_a) has a nearly flat potential then it would have increased with time as soon as the coherent axion oscillations commenced. As a result of this, energy would have drained out of the axion oscillations into the nearly massless radial mode, and from thence been redshifted away by the cosmic expansion. We find that the initial value of f_a can be as large as 10^{16} GeV. In the course of cosmic evolution this would have “relaxed” to a value we presume to be near the Planck scale. Axions in this scenario would therefore be extremely hard if not impossible to detect. However, the radial mode may play the role of “quintessence”.⁴

The plan of the paper is as follows. In section 2, a rough sketch of the axion energy problem in conventional axion models is given. In section 3, a simplified discussion of the relaxing axion scenario is given to show how energy is drained out of the axion field. In section 4, a more detailed quantitative analysis is given, still assuming, however, that the radial direction is exactly flat. In section 5, the radial potential is discussed, in particular how flat it must be and how such flatness might arise.

2 The usual axion energy problem

In ordinary axion models, the Peccei-Quinn symmetry is spontaneously broken by a complex scalar field, which we shall denote Φ . The expectation value

of Φ is called f_a , while the phase of Φ is the axion mode. Thus one may write $\Phi = r e^{i\theta}$, where $\langle r \rangle = f_a$, and the axion field a is given by $a = f_a \theta$. Since the Peccei-Quinn symmetry is anomalous, QCD instanton effects give the axion a mass. The instanton-generated potential has the form $V_I = -\mu_0^4 \cos(N\theta)$, where μ_0 is of order the QCD scale, and N is an integer, which for present purposes can be taken to be 1.

When the temperature in the early universe was well above the QCD scale the instanton-generated potential was not fully turned on, whereas after the temperature fell below the QCD scale it had essentially its full zero-temperature strength. Let us call the time when the instanton potential reached nearly full strength, t_0 . Since the temperature was then of order μ_0 , $t_0 \sim M_P/\mu_0^2$. Assuming a prior epoch of inflation, the axion field at t_0 was approximately spatially constant, but barring an extremely unlikely coincidence, it had no reason to be sitting at the minimum of its potential (which we take to be at $\theta = 0$). Rather, it had some arbitrary value $\theta(t_0) = \theta_0$. Consequently, the axion field underwent coherent oscillations about its minimum. The energy density in these oscillations at time t_0 would have been approximately $\frac{1}{2}\mu_0^4\theta_0^2$. This energy scaled as R^{-3} , where R is the scale factor of the universe, and therefore the energy density in the coherent axion oscillations remained (after t_0) in a constant ratio to the baryon energy density. Since at t_0 the baryon energy density was of order $10^{-10}m_p\mu_0^3$, the present ratio of axion to baryon energy is roughly $(\frac{1}{2}\theta_0^2\mu_0^4)/(10^{-10}m_p\mu_0^3) \sim 10^9\theta_0^2$. If the axions are not to overclose the universe, $\theta_0^2 \lesssim 10^{-7}$.

This bound on θ_0 can be satisfied if f_a is sufficiently small. The point is that the instanton-generated potential of the axion field did not turn on instantaneously. If the potential turned on slowly then the number of quanta in the coherent axion oscillations remained constant since it is an adiabatic invariant. This implies that as the mass of the axion increased the amplitude of the oscillations decreased, and therefore by t_0 , when the potential reached full strength, the amplitude θ_0 could have been very small. A simple estimate gives that $\theta_0^2 \sim f_a/M_P$. Thus, to satisfy the bound on ρ_{axion} one requires that $f_a \lesssim 10^{12}$ GeV.

3 The relaxing axion mechanism

Let us suppose that the field Φ whose phase is the axion has a flat potential in the radial direction. Then the Lagrangian density can be written

$$\begin{aligned}
L &= \frac{1}{2} |\partial_\mu \Phi|^2 + \mu_0^4 \cos \theta, \\
&\cong \frac{1}{2} (\partial_\mu r)^2 + \frac{1}{2} r^2 (\partial_\mu \theta)^2 - \frac{1}{2} \mu_0^4 \theta^2.
\end{aligned} \tag{1}$$

As before, $\Phi = r e^{i\theta}$, and the axion field is $a = r\theta$. Ignoring spatial derivatives of the fields, the equations of motion of r and θ are

$$\begin{aligned}
0 &= \ddot{r} + 3H\dot{r} - r\dot{\theta}^2, \\
0 &= \ddot{\theta} + (3H + 2\frac{\dot{r}}{r})\dot{\theta} + \mu_0^4 r^{-2} \theta.
\end{aligned} \tag{2}$$

The last term in the equation for the radial mode r is just the centrifugal force, and it is this that drives r to larger values as θ oscillates.

The energy densities in the radial and axion modes are given simply by $\rho_r = \frac{1}{2}\dot{r}^2$ and $\rho_\theta = \frac{1}{2}r^2\dot{\theta}^2 + \frac{1}{2}\mu_0^4\theta^2$. The energies in a comoving volume in these modes are given by $E_r = R^3\rho_r$ and $E_\theta = R^3\rho_\theta$. Using the equation of motion of θ to eliminate $\ddot{\theta}$, it is easy to show that

$$\dot{E}_\theta = R^3 \left[-\frac{\dot{r}}{r} r^2 \dot{\theta}^2 + 3H \left(-\frac{1}{2} r^2 \dot{\theta}^2 + \frac{1}{2} \mu_0^4 \theta^2 \right) \right]. \tag{3}$$

If the oscillator parameters evolve adiabatically, then averaged over many oscillations the kinetic and potential energy in the oscillator should be equal. That is, $\langle \frac{1}{2} r^2 \dot{\theta}^2 \rangle = \langle \frac{1}{2} \mu_0^4 \theta^2 \rangle = \frac{1}{2} \langle \rho_\theta \rangle$. Therefore, averaged over many oscillations,

$$\dot{E}_\theta = -\frac{\dot{r}}{r} E_\theta \tag{4}$$

In a similar way, using the equation of motion of r to eliminate \ddot{r} , and averaging over many oscillations, one finds that

$$\dot{E}_r = \frac{\dot{r}}{r} E_\theta - 3H E_r. \tag{5}$$

The interpretation is clear. The increase in r caused by the centrifugal force drains energy out of the axion oscillations and into the radial mode, while at the same time energy is redshifted away from the radial mode. To put it another way, as r increases the effective mass of the axions, μ_0^2/r , decreases, while the number of axions remains constant. Thus $E_\theta \sim 1/r$, as implied also by Eq. 4.

Suppose that $r \sim t^q$, and $E_\theta \sim t^{-q}$. The exponent q can be determined by writing the equation of motion of r in terms of E_θ as follows: $0 = \ddot{r} + 3H\dot{r} - R^{-3}\langle E_\theta \rangle r^{-1}$. The first two terms go as $t^{(q-2)}$, while the last goes as $t^{-(2q+3/2)}$, assuming that $R \sim t^{1/2}$. Therefore $q = 1/6$. Writing $E_\theta = E_{\theta 0}(t/t_0)^{-1/6}$, Eq. 5 is solved by $E_r = \frac{1}{8}E_{\theta 0}(t/t_0)^{-1/6} + ct^{-3/2}$. Thus, for large t , the radial energy is one-eighth of the energy in the coherent axion oscillations, and the energy in a comoving volume in either mode falls off as $t^{-1/6}$.

4 A more detailed analysis

So far we have not taken into account the temperature dependence of the instanton-generated potential of the axion field. According to the dilute-instanton-gas calculation of Gross, Pisarski, and Yaffe,⁵ the instanton potential for θ at high temperature goes as $\mu^4 \sim T^4 \exp(-8\pi^2/g^2(T)) \sim T^4 \exp(\frac{1}{3}(11N - 2N_f) \ln T) \sim T^{(-7+2N_f/3)}$. We shall assume henceforth that $\mu^4 = \mu_0^4(T_0/T)^{2k} = \mu_0^4(t/t_0)^k$, for $T \gg \mu_0$ (i.e. $t \ll t_0$), and that $\mu^4 \cong \mu_0^4$ for $t \gg t_0$. The dilute-instanton-gas calculation suggests that $k \cong 5/2$, but we shall keep k as a parameter.

If we change variables to $\tau \equiv t/t_0$, and denote $\partial/\partial\tau$ by a dot, then we can write the equations of motion

$$\begin{aligned} 0 &= \ddot{r} + \frac{3}{2\tau}\dot{r} - r\dot{\theta}^2, \\ 0 &= \ddot{\theta} + (\frac{3}{2\tau} + 2\frac{\dot{r}}{r})\dot{\theta} + (\mu_0^4 t_0^2/r^2)\tau^k \theta. \end{aligned} \tag{6}$$

The effective mass of the field θ , then, is $m_\theta(\tau) = (\mu_0^2 t_0/r(\tau))\tau^{k/2}$. Therefore it is reasonable to make the ansatz that $\theta(\tau)$ has the form

$$\theta(\tau) = A(\tau)e^{iB(\tau)}, \tag{7}$$

where A and B are real and

$$\dot{B}(\tau) = m_\theta(\tau) = (\mu_0^2 t_0/r(\tau))\tau^{k/2}. \tag{8}$$

Substituting into the equation of motion for θ and taking the real and imaginary parts of the equation, one gets (using the fact that $\dot{r}/r = k/2\tau - \dot{B}/\dot{B}$)

$$\begin{aligned}
0 &= \ddot{A} + \left[\left(\frac{3}{2} + k \right) / \tau - 2\ddot{B}/\dot{B} \right] \dot{A}, \\
0 &= -\ddot{B} + \left[\left(\frac{3}{2} + k \right) / \tau + 2\dot{A}/A \right] \dot{B}.
\end{aligned} \tag{9}$$

The first of these equations is exactly solved by $(\dot{A}/\dot{B}^2)\tau^{(3/2+k)} = \text{constant}$, while the second is solved exactly by $(\dot{B}/A^2)\tau^{-(3/2+k)} = \text{constant}$. Eliminating \dot{B} from these equations and solving gives

$$\begin{aligned}
A &= \theta_0(\tau^{(5/2+k)} + C))^{-1/3}, \\
\dot{B} &= c'\theta_0^2(\tau^{(5/2+k)} + C)^{-2/3}\tau^{(3/2+k)},
\end{aligned} \tag{10}$$

where c' , C , and θ_0 are integration constants. After many oscillations the integration constant C can be neglected, and one has, using Eqs. 7 and 10, a solution for $\theta(\tau)$ of the following form:

$$\theta(\tau) = \theta_0\tau^{-(5/6+k/3)} \exp(ib_0\tau^{(5/6+k/3)} + ib'_0). \tag{11}$$

($b_0 = \frac{c'\theta_0^2}{5/6+k/3}$.) Eqs. 8 and 10 directly give the solution for the radial mode: $r(\tau) = \frac{\mu_0^2 t_0}{(5/6+k/3)b_0} \tau^{(1/6+k/6)}$ (again, neglecting the integration constant C). Note that for the case of $k = 0$, which corresponds to a fixed value of μ^4 , the radial variable increases as $\tau^{1/6}$, in agreement with the result obtained in section 3.

It must be checked that this solution for $r(\tau)$ and the solution for $\theta(\tau)$ given in Eq. 11 satisfy the equation of motion of r . Assuming that $\dot{\theta}$ is dominated by the rapid oscillations of θ rather than by the slow variation of its amplitude, one has that $\dot{\theta} \cong ib_0(5/6 + k/3)\theta_0\tau^{-1} \exp(ib_0\tau^{(5/6+k/3)} + ib'_0)$. Averaged over many oscillations, therefore, $\langle \dot{\theta}^2 \rangle = \frac{1}{2}b_0^2(5/6 + k/3)^2\theta_0^2\tau^{-2}$. Substituting this and the expression for $r(\tau)$ into the equation of motion for r (see Eq. 6), one sees that that equation is satisfied if $b_0 = \frac{\sqrt{(1+k)(4+k)}}{5+2k}\theta_0^{-1}$. Thus we have that

$$r(\tau) = \frac{6\mu_0^2 t_0}{\sqrt{(1+k)(4+k)}} \theta_0 \tau^{(1/6+k/6)}. \tag{12}$$

The solutions given in Eqs. 11 and 12 apply to the period $\tau < 1$, in which the instanton potential was still turning on. The same expressions with k set to zero apply to the period $\tau > 1$, after the instanton potential turned on. The true solution will smoothly interpolate between these in the period $\tau \sim 1$. (Note that the $k \neq 0$ and $k = 0$ solutions for θ actually agree at $\tau = 1$, while the $k \neq 0$ and $k = 0$ solutions for r differ at $\tau = 1$ by a factor of $\sqrt{(1+k)(4+k)}/2 \approx 2.4$.)

One is now able to estimate the energy in the axion oscillations. The crucial parameter is the value of r at the time when the axion oscillations started. We will call that time t_i . The oscillations started when the effective mass of the θ field, $m_\theta = \dot{B}$, became equal to the expansion rate of the universe, $H = (2t)^{-1}$. That is, when $(2t_i)^{-1} \approx (\mu_0^2 t_0 / r(t_i))(t_i/t_0)^{k/2}$. Clearly, the smaller $r(t_i)$ was, the earlier the axion oscillations began. Turning this around, $r(t_i) \approx 2\mu_0^2 t_0 (t_i/t_0)^{(1+k/2)} \sim M_P (t_i/t_0)^{(1+k/2)}$.

At t_i it is to be expected that θ was of order unity, since it had not had time to be affected by the instanton potential. But according to Eq. 11, $\theta(t_i) \sim \theta_0 (t_i/t_0)^{-(5/6+k/3)}$. Thus, θ_0 is of order $(t_i/t_0)^{(5/6+k/3)}$, or, in terms of $r(t_i)$,

$$\theta_0 \sim (r(t_i)/M_P)^{\frac{5+2k}{6+3k}}. \quad (13)$$

As was seen in section 2, the factor θ_0^2 tells how much the energy of the coherent axion oscillations was suppressed by the time the axion potential fully turned on at t_0 . We will call this suppression factor S_{before} .

$$S_{before} \approx \theta_0^2 \sim (r(t_i)/M_P)^{\frac{2}{3}(\frac{5+2k}{2+k})}. \quad (14)$$

If this were the only suppression of the axion energy, solving the axion energy problem would require that $\theta_0 \lesssim 10^{-7/2}$. With $k = 5/2$ this gives $r(t_i) \lesssim 2 \times 10^{14}$ GeV. That is, the initial value of “ f_a ” can have been quite near the grand unification scale. By t_0 this would have increased, according to Eq. 12, to a value $r(t_0) \sim \mu_0^2 t_0 \theta_0 \sim \theta_0 M_P \sim 3 \times 10^{15}$ GeV. One possibility, which we will call Case I, is that the radial mode stopped evolving at that point, because its potential has a minimum there. Another possibility, which we will call Case II, is that r continued to increase to some final value near the Planck scale. That would mean that the axion energy would have been further suppressed by the evolution of r in the period $t > t_0$. Since $r(t_0) \sim \theta_0 M_P$, and it is assumed that $r(t_f) \sim M_P$, there is an increase of r by a factor of θ_0^{-1} in this period. It is easily shown that the energy of the axion field in a

comoving volume varies inversely with r in this period (as was seen already in section 3), so that the further suppression of the axion energy, which we shall call S_{after} , is given by

$$S_{after} \approx \theta_0, \quad (15)$$

or

$$S_{total} = S_{before} S_{after} \approx \theta_0^3. \quad (16)$$

In Case II, therefore, it is only necessary that $\theta_0 \lesssim 10^{-7/3}$, meaning that $r(t_i) \lesssim 10^{16}$ GeV. Case I and Case II are, in a sense, the extreme cases. One can consider intermediate cases as well. But one sees that in general the relaxing axion scenario would have f_a starting out in the range 10^{14} to 10^{16} GeV, near the grand unification scale, and evolving to higher values.

5 The flatness of the radial potential

The mechanism described in the preceding sections depends crucially on the assumption that the potential in the radial direction is nearly flat. For the mechanism to work, the centrifugal term in the equation of motion for r had to have dominated over the force coming from the potential energy of r . That is, $r\dot{\theta}^2 > |V'(r)|$. One can write this as $\rho_{axion} > |rV'(r)|$, where ρ_{axion} is the energy in the coherent axion oscillations.

At some point this condition was no longer satisfied and the radial field's evolution was controlled by $V(r)$. Unless the radial field was at that point overdamped (not so in the cases of interest) it would have started to oscillate about the minimum of $V(r)$. For reasonable potentials (where one assumes that the cosmological constant problem has somehow been solved) one would expect that $V(r) \sim |rV'(r)|$, and therefore the energy in the coherent oscillations of the radial mode were also of that order. Consequently, when the coherent radial oscillations began, the energy in them was typically of the same order as the energy in the coherent axion oscillations. Thus the coherent radial oscillations do not in themselves pose a cosmological problem.

However, for the mechanism to work at all it is necessary that the centrifugal term in the equation of motion of r dominated over the potential term for a sufficiently long time to solve the axion energy problem. This puts a constraint on the flatness of $V(r)$.

In Case I, it is assumed that the centrifugal term drove r until t_0 , when the temperature was of order μ_0 . At that time ρ_{axion} had to have been less than about $10^2 \rho_B \sim 10^{-8} \mu_0^3 m_p \sim 10^{-10} \text{GeV}^4$. Thus, at t_0 it must also have been that $|rV'(r)| \lesssim 10^{-10} \text{GeV}^4$.

In Case II, the centrifugal term is assumed to have dominated until r got to be of order M_P . Since $r(t_0) \sim \theta_0 M_P$, and r grew as $t^{1/6}$ for $t > t_0$, this happened at a time $t_f \sim \theta_0^{-6} t_0$. Assuming a radiation dominated universe, $T(t_f) \sim \theta_0^3 T(t_0) \sim S_{total} \mu_0 \sim 10^{-7} \mu_0$. Therefore, at t_f it must have been that $|rV'(r)| \lesssim 10^{-8} T(t_f)^3 m_p \sim 10^{-31} \text{GeV}^4$.

How can the potential be that flat? Certainly it is trivial to arrange that the radial direction be flat in the supersymmetric limit. The real problem is to insulate the radial mode from supersymmetry breaking. This is easiest to do if supersymmetry is broken at low energies, as it is in models with gauge-mediated supersymmetry breaking.⁶ In such a model, the ordinary quarks must be split from their supersymmetry partners by an amount that is of order 10^2 to 10^3 GeV, which scale we will call m_0 . Since the axion sector must couple directly or indirectly to the quark sector for the Peccei-Quinn symmetry to have a QCD anomaly, supersymmetry breaking will be fed into the radial mode of the axion sector through quark loops. Typically, then, the radial mode of the axion sector will acquire a potential of the form $\epsilon m_0^4 \ln(r/m_0)$. ϵ is model-dependent and is smaller the more indirectly and the more weakly the axion sector couples to the quark sector. In Case I, one has that $\epsilon m_0^4 \lesssim 10^{-10} \text{GeV}^4$, implying (if $m_0 \sim 1$ TeV) that $\epsilon \lesssim 10^{-22}$. In Case II, one has the more severe constraint that $\epsilon \lesssim 10^{-43}$.

To see how such small values of ϵ might be achieved, consider first a conventional axion model where the radial mode has a tree-level potential. The relevant terms in the superpotential would have the following general structure:

$$W_{axion} = g S \bar{Q} Q + W_S, \quad (17)$$

where Q and \bar{Q} are lefthanded quark and anti-quark superfields, and S is a superfield containing the axion. W_S is some set of terms, generally involving other fields, which has the effect of fixing $|\langle S \rangle|$ to have some value M . The phase of S is, however, assumed not to be fixed except by QCD instanton effects. One can write $S = (M + \tilde{S}) e^{i\theta}$, where here we mean by S the bosonic component, and the axion field is $M\theta$. The radial mode \tilde{S} would typically have some mass of order M .

Consider, now, a somewhat different model, with the corresponding terms in the superpotential being the following

$$W_{axion} = gS\overline{Q}Q + W_S + g'(SA - M'B)Y, \quad (18)$$

where S , A , B , and Y are all gauge singlets, and as before W_S has the effect of making $|\langle S \rangle| = M$ but leaving the phase of S undetermined. Suppose that both M and M' are very large compared to the scale of supersymmetry breaking. It is apparent that, since S has a Peccei-Quinn charge, so must either A or B . Therefore, if $\langle A \rangle$ and $\langle B \rangle$ are larger than M , they rather than $\langle S \rangle$ control the value of f_a .

Assume that Y has no other couplings in the superpotential. Then one of the terms in the scalar potential is $|g'(SA - M'B)|^2$. This term fixes only the ratio of A and B and leaves them otherwise undetermined. There is, therefore, a direction that is flat in the supersymmetric limit along which A and B can become arbitrarily large. This flat direction would play the role of r in this model.

Writing $S = (M + \tilde{S})e^{i\theta}$, $A = re^{i\alpha}$, and $B = (\frac{M}{M'}r + \tilde{B})e^{i(\alpha+\theta+\delta)}$, the aforementioned term can easily be found to give a mass to the fields \tilde{S} , \tilde{B} , and $\tilde{\delta} \equiv r\delta$. In particular, one finds that $|g'(SA - M'B)|^2 = g'^2 M^2 \tilde{\delta}^2 + g'^2 (\langle r \rangle \tilde{S} - M' \tilde{B})^2 +$ terms of cubic and higher order. In addition, there is the mass term for \tilde{S} coming from W_S . The phase α is an exact goldstone mode, while θ , since it corresponds to an anomalous $U(1)$ by virtue of its coupling to the quarks, is the axion mode. The scalar field r has (before supersymmetry breaking) a flat potential. It is easily seen that the Lagrangian terms for r and θ have (after suitable rescaling of fields) essentially the same form as those shown in Eq. 1. Thus this model is an implementation of the relaxing axion mechanism. The question is how large a potential r gets in this model. If supersymmetry breaking is mediated from some hidden sector by Standard Model gauge interactions, then only \overline{Q} and Q will directly feel it. The splittings in the supermultiplet S will only arise through a quark loop, and the splittings in the multiplets A and B will only arise through diagrams involving both an S loop and a quark loop. One would expect that $\epsilon \sim (g^2/16\pi^2)(g'^2/16\pi^2)$. This can easily be as small as required. For example, if M and M' are near the unification scale, the coupling g could even be as small as 10^{-14} .

It seems to be possible to shield the radial mode from supersymmetry breaking even more thoroughly. As an extreme case one could imagine a

whole series of gauge-singlet sectors separating the quark sector from the axion sector: $S_{quark} \longleftrightarrow S_1 \longleftrightarrow \dots \longleftrightarrow S_n \longleftrightarrow S_{axion}$, where the S 's denote various sectors, and the arrows represent a coupling in the superpotential between two sectors. In this case, supersymmetry breaking in the radial mode would be an n -loop effect. On the other hand, because of the couplings between sectors, there are fields in each sector that have non-trivial Peccei-Quinn charges. Thus the axion field, though removed by several steps from the quarks, will nonetheless get a potential from QCD instanton effects that goes as $-\mu_0^4 \cos(N\theta)$, where N is an integer.

From these examples, it seems that there is no reason in principle why the radial mode could not be sufficiently flat to allow the relaxing axion mechanism to work. It is an interesting question whether this flat radial direction can be identified with other flat directions that have been discussed in recent years. Could it be, to mention two obvious examples, the dilaton of superstring theory or the “quintessence” field? In any event, given the increasing role that very flat potentials have played in particle physics and cosmology (inflavons, quintessence, dilatons, moduli, and the axion itself), it is suggestive that a flat radial direction can allow the breaking of the Peccei-Quinn symmetry to take place at the unification scale or even higher.

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